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Formulary

Superconductivity for Engineers

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Material properties:

$$\begin{aligned}
\text{Niobium} \quad T_\mathrm{c} &= 9.25\,\mathrm{K} \\
B_\mathrm{c} &= 206\,\mathrm{mT} \\
\lambda_\mathrm{L}(0) &= 52\,\mathrm{nm} \\
\rho &= 1.52 \cdot 10^{-7}\,\Omega\mathrm{m} \\
\text{Lead} \quad T_\mathrm{c} &= 7.20\,\mathrm{K} \\
B_\mathrm{c} &= 80.3\,\mathrm{mT} \\
\lambda_\mathrm{L}(0) &= 39\,\mathrm{nm} \\
\rho &= 2.1 \cdot 10^{-7}\,\Omega\mathrm{m} \\
\text{Tin} \quad T_\mathrm{c} &= 3.72\,\mathrm{K} \\
B_\mathrm{c} &= 30.6\,\mathrm{mT} \\
\lambda_\mathrm{L}(0) &= 42\,\mathrm{nm} \\
\rho &= 1.15 \cdot 10^{-7}\,\Omega\mathrm{m} \\
\text{Aluminium} \quad T_\mathrm{c} &= 1.18\,\mathrm{K} \\
B_\mathrm{c} &= 10.5\,\mathrm{mT} \\
\lambda_\mathrm{L}(0) &= 45\,\mathrm{nm} \\
\rho &= 2.65 \cdot 10^{-8}\,\Omega\mathrm{m} \\
\text{Titanium} \quad T_\mathrm{c} &= 0.40\,\mathrm{K} \\
B_\mathrm{c} &= 5.6\,\mathrm{mT} \\
\rho &= 4.0 \cdot 10^{-7}\,\Omega\mathrm{m} \\
\text{Beryllium} \quad T_\mathrm{c} &= 26\,\mathrm{mK} \\
B_\mathrm{c} &= 0.11\,\mathrm{mT} \\
\rho &= 3.6 \cdot 10^{-8}\,\Omega\mathrm{m} \\
\text{Constantan} \quad \rho &= 4.9 \cdot 10^{-7}\,\Omega\mathrm{m} \\
\text{Platinum} \quad \rho &= 1.1 \cdot 10^{-7}\,\Omega\mathrm{m}
\end{aligned}$$

	Al	Pb	Nb	Pt
Crystal structure	fcc	fcc	bcc	fcc
Lattice constant (Å)	4.046	4.920	3.3008	3.912

Crystal structure and lattice constant of various materials

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Physical constants

Avogadro constant	$N_A = 6.02 \cdot 10^{23}\,\mathrm{mol}^{-1}$
Boltzmann constant	$k_B = 1.38 \cdot 10^{-23}\,\mathrm{J/K}$
Electron charge	$e = 1.60 \cdot 10^{-19}\,\mathrm{C}$
Electron mass	$m_e = 9.11 \cdot 10^{-31}\,\mathrm{kg}$
Magnetic flux quantum	$\Phi_0 = 2.07 \cdot 10^{-15}\,\mathrm{Vs}$
Planck constant	$h = 6.63 \cdot 10^{-34}\,\mathrm{Js}$
Vacuum permeability	$\mu_0 = 1.26 \cdot 10^{-6}\,\mathrm{N/A^2}$
Vacuum permittivity	$\epsilon_0 = 8.85 \cdot 10^{-12}\,\mathrm{F/m}$
Vacuum speed of light	$c = 2.99 \cdot 10^{8}\,\mathrm{m/s}$

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Normal metals and properties of the normal conducting state

Fermi vector, Fermi velocity, Fermi energy, Fermi temperature and density of states per volume:

$$k_\mathrm{F} = \left(3\pi^2 n_\mathrm{el}\right)^{1/3}$$
$$v_\mathrm{F} = \frac{\hbar}{m_\mathrm{e}}\left(3\pi^2 n_\mathrm{el}\right)^{1/3}$$
$$E_\mathrm{F} = \frac{\hbar^2 k_\mathrm{F}^2}{2m_\mathrm{e}}$$
$$T_\mathrm{F} = \frac{E_\mathrm{F}}{k_\mathrm{B}}$$
$$D(E) = \frac{(2m_\mathrm{e})^{3/2}\sqrt{E}}{2\pi^2\hbar^3}$$

Fermi-Dirac-distribution

$$f(E,T) = \frac{1}{\exp\left(\frac{E-E_\mathrm{F}}{k_\mathrm{B}T}\right)+1}$$

Matthiesen rule

$$\rho = \rho_1 + \rho_2 + \dots$$
$$\frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \dots$$

Electrical conductance, heat conductance, Wiedemann-Franz law

$$\sigma = \frac{n_\mathrm{el}e^2\tau}{m_\mathrm{e}} = \frac{n_\mathrm{el}e^2 l}{m_\mathrm{e}v_\mathrm{F}}$$
$$k = \frac{\pi^2}{3}\frac{n k_\mathrm{B}^2\tau}{m}T$$
$$\frac{\sigma}{k} = \frac{\pi^2}{3}\left(\frac{k_\mathrm{B}}{e}\right)^2 T$$

Residual-resistance ratio

$$RRR = \frac{\rho(300\,\mathrm{K})}{\rho(4.2\,\mathrm{K})}$$

Current decay within normal conducting closed loop

$$I(t) = I_0\,\mathrm{e}^{-Rt/L}$$

Perfect conductor, ideal diamagnetism, Two-Fluid-Model, London theory

Maxwell equations

$$\nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon_0}$$
$$\nabla\cdot\mathbf{B} = 0$$
$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$
$$\nabla\times\mathbf{B} = \mu_0\left(\mathbf{J} + \epsilon_0\frac{\partial\mathbf{E}}{\partial t}\right)$$

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London equations

$$\frac{\partial}{\partial t}(\Lambda \mathbf{j}_s) = \mathbf{E}$$
$$\nabla \times (\Lambda \mathbf{j}_s) + \mathbf{B} = 0$$
$$\Lambda = \frac{m_s}{n_s q_s^2} = \mu_0 \lambda_L^2$$

Spatial dependence of magnetic field

$$\nabla^2 \mathbf{B} - \frac{1}{\lambda_L^2}\mathbf{B} = 0$$

London penetration depth

$$\lambda_L(T) = \sqrt{\frac{m_s}{\mu_0 n_s(T) q_s^2}}$$

Magnetic flux

$$\Phi = \int_A B \cdot \mathrm{d}A \quad (\text{special case}: \ \Phi = B_\perp A)$$
$$\Phi = LI$$

Superconducting wire with radius $R \gg \lambda_L$ and total current I

$$j(r) = \frac{I}{2\pi R \lambda_L} \exp\left[-\frac{R-r}{\lambda_L}\right]$$
$$B(r) = \frac{\mu_0 I}{2\pi R} \exp\left[-\frac{R-r}{\lambda_L}\right] \quad \text{for} \quad R < r$$
$$I = 2\pi R \lambda_L j_0$$

Temperature dependencies

$$B_c(T) = B_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$
$$\lambda_L(T) = \lambda_L(0)\left[1 - \left(\frac{T}{T_c}\right)^4\right]^{-1/2}$$
$$j_c(T) = j_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]\left[1 - \left(\frac{T}{T_c}\right)^4\right]^{1/2}$$

Temperature dependencies for $T \to T_c$

$$B_c(T) \approx 2B_c(0)\left[1 - \frac{T}{T_c}\right]$$
$$\lambda_L(T) \approx \frac{1}{2}\lambda_L(0)\left[1 - \frac{T}{T_c}\right]^{-1/2}$$
$$j_c(T) \approx 4j_c(0)\left[1 - \frac{T}{T_c}\right]^{3/2}$$

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BCS theory

Isotope effect

$$T_c \propto 1/\sqrt{M}$$

Coherence length

$$\xi_{BCS} = \frac{\hbar v_F}{\pi \Delta(0)}$$

Condensation energy per volume

$$E_{cond}/V = \frac{1}{4} D(E_F)\Delta(0)^2$$

Energy gap

$$\Delta(0) = 1.76\, k_B T_c$$

Temperature dependence of energy gap for $T \to T_c$

$$\frac{\Delta(T)}{\Delta(0)} = 1.74\sqrt{1 - \frac{T}{T_c}}$$

Disordered superconductors, Pippard theory, microwave properties

Penetration depths in superconductors

$$\xi_0 \ll \lambda_L : \lambda \approx \lambda_L$$
$$\xi_0 \gg \lambda_L : \lambda \approx \left(\lambda_L^2 \xi_0\right)^{1/3}$$
$$l \ll \xi_0 : \lambda \approx \lambda_L \sqrt{\xi_0 / l}$$

Pippard length

$$\xi_P^{-1} = \xi_0^{-1} + l^{-1}$$

Superconductor surface resistance

$$R_S = \frac{1}{2}\omega^2 \mu_0^2 \lambda_L^3 \sigma_0 \frac{n_n}{n}$$

Superconductor surface reactance

$$X_S = \omega \mu_0 \lambda_L$$

Josephson Junctions and SQUIDs

1. Josephson equation

$$j_s(\varphi) = j_c \sin(\varphi)$$

2. Josephson equation

$$\dot{\varphi} = \frac{2\pi}{\Phi_0} V$$

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Stewart-McCumber-parameter

$$\beta_c = \frac{2\pi I_c R^2 C}{\Phi_0}$$

Screening parameter

$$\beta_L = \frac{2 L I_c}{\Phi_0}$$

Voltage drop over dc-SQUID with $\beta_L \ll 1$ and $\beta_c \ll 1$

$$\langle V_{SQ}(t) \rangle = I_c R \sqrt{\left(\frac{I_{bias}}{2 I_c}\right)^2 - \cos^2\left(\pi \frac{\Phi_{ext}}{\Phi}\right)}$$

Type II superconductors

Critical magnetic fields

$$B_{c,th}(T) = \frac{\Phi_0}{2\pi\sqrt{2}\xi_{GL}(T)\lambda_L(T)}$$

$$B_{c,1} \approx \frac{\Phi_0}{4\pi\lambda_L^2} \ln\kappa$$

$$B_{c,2} = \frac{\Phi_0}{2\pi\xi_{GL}^2}$$

Periodic Table of the Elements

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 H 1.01																	2 He 4.00
3 Li 6.94	4 Be 9.01											5 B 10.81	6 C 12.01	7 N 14.01	8 O 16.00	9 F 19.00	10 Ne 20.18
11 Na 22.99	12 Mg 24.31											13 Al 26.98	14 Si 28.09	15 P 30.97	16 S 32.07	17 Cl 35.45	18 Ar 39.95
19 K 39.10	20 Ca 40.08	21 Sc 44.96	22 Ti 47.87	23 V 50.94	24 Cr 51.99	25 Mn 54.94	26 Fe 55.85	27 Co 58.93	28 Ni 58.69	29 Cu 63.55	30 Zn 65.38	31 Ga 69.72	32 Ge 72.63	33 As 74.92	34 Se 78.97	35 Br 79.90	36 Kr 83.80
37 Rb 85.47	38 Sr 87.62	39 Y 88.91	40 Zr 91.22	41 Nb 92.91	42 Mo 95.95	43 Tc 98.91	44 Ru 101.07	45 Rh 102.91	46 Pd 106.42	47 Ag 107.87	48 Cd 112.41	49 In 114.82	50 Sn 118.71	51 Sb 121.76	52 Te 127.6	53 I 126.90	54 Xe 131.29
55 Cs 132.91	56 Ba 137.33	57-71	72 Hf 178.49	73 Ta 180.95	74 W 183.84	75 Re 186.21	76 Os 190.23	77 Ir 192.22	78 Pt 195.09	79 Au 196.97	80 Hg 200.59	81 Tl 204.38	82 Pb 207.2	83 Bi 208.98	84 Po [208.98]	85 At 209.99	86 Rn 222.02
87 Fr 223.02	88 Ra 226.03	89-103	104 Rf [261]	105 Db [262]	106 Sg [266]	107 Bh [264]	108 Hs [269]	109 Mt [278]	110 Ds [281]	111 Rg [280]	112 Cn [285]	113 Nh [286]	114 Fl [289]	115 Mc [289]	116 Lv [293]	117 Ts [294]	118 Og [294]

57 La 138.91	58 Ce 140.12	59 Pr 140.91	60 Nd 144.24	61 Pm 144.91	62 Sm 150.36	63 Eu 151.96	64 Gd 157.25	65 Tb 158.93	66 Dy 162.50	67 Ho 164.93	68 Er 167.26	69 Tm 168.93	70 Yb 173.06	71 Lu 174.97
89 Ac 227.03	90 Th 232.04	91 Pa 231.04	92 U 238.03	93 Np 237.05	94 Pu 244.06	95 Am 243.06	96 Cm 247.07	97 Bk 247.07	98 Cf 251.08	99 Es [254]	100 Fm 257.10	101 Md 258.1	102 No 259.10	103 Lr [262]

Periodic table block	Periodic table group	Valence electrons
s	Group 1 (alkali metals)	1
	Group 1 (alkaline earth metals) and helium	2
f	Lanthanides and actinides	3-16 [1]
d	Group 3-12 (transition metals)	3-12 [2]
p	Group 13 (boron group)	3
	Group 14 (carbon group)	4
	Group 15 (pnictogens)	5
	Group 16 (chalcogens)	6
	Group 17 (halogens)	7
	Group 18 (noble gases) except helium	8

1) Consists of ns, (n-2)f and (n-1)d electrons.
2) Consists of ns and (n-1)d electrons.